DOCUMENT RESUME

ED 168 836 SE 026 914

AUTHOR Stengel, Arthur; And Others
TITLE Mathematical Problem Solving Project Technical Report II: D. Learning to Solve Problems by Solving Problems: A Report of a Preliminary Investigation. Final Report.
INSTITUTION Indiana Univ., Bloomington. Mathematics Education Development Center.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE May 77
GRANT NSF-PES-74-15045
NOTE 54p.; For related documents, see SE 026 911-934

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Concept Formation; Elementary Education; *Elementary School Mathematics; *Instruction; Learning Activities; *Mathematics Education; Pilot Projects; *Problem Solving; *Research; *Teaching Techniques
IDENTIFIERS *Mathematical Problem Solving Project

ABSTRACT

The activities described in this report are focused on a preliminary investigation of the effects of presenting problems to children without any prior formal problem-solving instruction, on the children's ability to solve the problems, and on their subsequent problem-solving performances. The work was an outgrowth of activities related to the observation of children's problem-solving efforts and the resulting conjectures. A description is given of the preparation of materials and the ensuing pilot study. Implications are discussed and some conjectures related. (MP)

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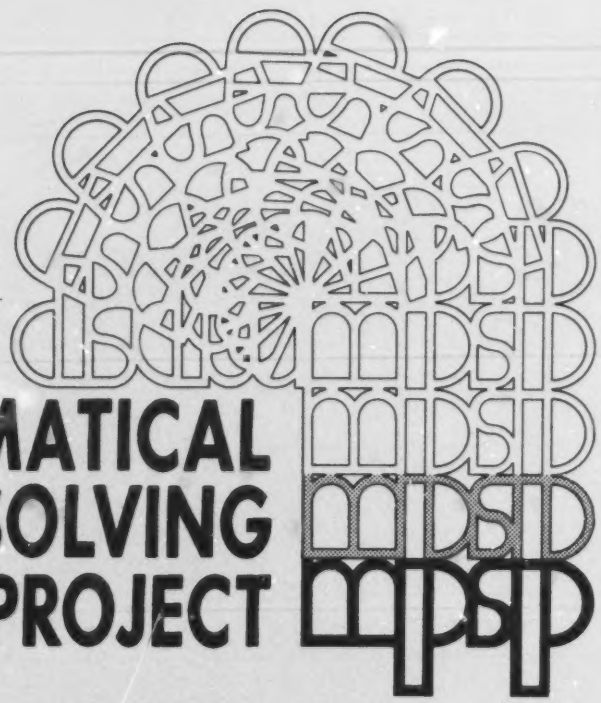

MATHEMATICAL PROBLEM SOLVING PROJECT

SE 026 914

A Project of the
MATHEMATICS EDUCATION DEVELOPMENT CENTER
Project Supported by
National Science Foundation Grant PES74-15045

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FINAL REPORT
MATHEMATICAL PROBLEM SOLVING PROJECT
TECHNICAL REPORT II:
D. LEARNING TO SOLVE PROBLEMS BY SOLVING PROBLEMS

A REPORT OF A PRELIMINARY INVESTIGATION

Report Prepared By

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May 1977

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A. *Introduction*

This report focuses on a specific effort of the Mathematical Problem Solving Project (MPSP) directed at presenting process problems to children and observing their performances as they solved these problems. This effort was carried out by staff members at Indiana University with children in the Bloomington, Indiana schools from September 1975 to May 1976. This work on process problem solving was an outgrowth of the first year's (September 1974 - May 1975) activities of MPSP related to the observation of children's problem solving efforts and the resulting conjectures. Further, the effort was enhanced by the widely held belief that a good way to improve children's problem solving performance is to simply have them solve (and then analyze) problems.

Learning by doing is one of the more commonly mentioned maxims of educational thought. As it relates specifically to mathematical problem solving, the idea that solving problems will improve problem-solving performance has been frequently hypothesized in the literature of mathematics education. The activities described in this report were focused on a preliminary investigation of the effects of presenting problems to children without any prior formal problem-solving instruction, on the children's ability to solve the problems and on their subsequent problem-solving performances.

In this preliminary investigation factors such as problems and problem types, problem-solving strategies, classroom delivery formats and other concerns were to be considered in anticipation of a later and more formal investigation (not funded) of this central hypothesis. A discussion of some of the above mentioned factors follows.

Problems and problem types. As reported in the 1975-76 MPSP proposal, a "problem" was conceptually defined "in terms of the individual" as follows: "A problem exists if he/she (the individual) desires to

obtain a goal but the path leading to the attainment of this goal is not immediately known and cannot be found by using habitual responses." It follows then, also in a generic sense, that the moment of the given problem's "solution" is defined in terms of the individual and can be viewed as the time at which the individual believes (s)he has achieved the desired goal of the problem. In essence, any statement of a problem may be viewed by various individuals differently creating, in one sense, as many problems as there are problem solvers. Additionally, most problems can be attacked in a variety of ways. Finally, the varying interpretations of a problem and the variety of avenues available for solution remain susceptible to the individual's sense of completion. An answer for one person may be totally unacceptable to another. It may be argued that commonly accepted definitions and conventions of mathematics and logic will substantially limit individual variations and in this way keep the size of the state space relatively small. This, however, assumes that the problem solver can employ common elements, and the developmental work of Piaget causes one to question the validity of such an assumption. One is left at this point with a framework which provides one starting point--the individual problem solver--from which cautious explorations can begin.

Another starting point is the nature of the problem types themselves. Specifically, problem statements were examined during the first year (1974-75) with the idea that general characteristics might be found which could be used in the selection and development of curriculum materials. This examination started with word problems which are typically found in mathematics textbooks. It was found that these problems were written to elicit use of one or more of the four basic operations. The problems, even those requiring use of more than one operation or the

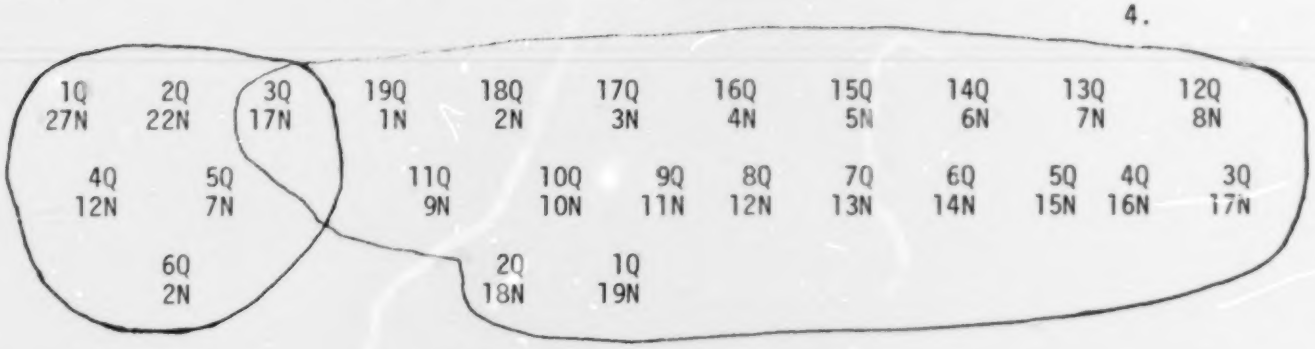
use of one operation more than once, required the manipulation of one condition at a time with a one-element solution set for each condition. A simple example may help:

> Example A: Apples cost 10¢ each. John bought 3 apples. Jane bought 4 apples. How much did John and Jane spend together?

For this problem, one may calculate John's expenditures, then Jane's and add them (3 x 10 + 4 x 10). Or, one may add the number of apples each person purchased and then calculate the total price [(3 + 4) x 10]. Even if different problem solvers performed the operations in a different order, the solution would be common to all. It should be obvious that this type of problem does not fit the conceptual definition of problem in that habitual responses are elicited. Further, the textbook problem is generally characterized by the fact that it can be solved by direct application of an algorithm(s). Another type of problem, the so-called process problem, was considered and used in the initial investigation. The process problem is characterized by the features that it cannot be solved using a simple algorithm (or at least not using one known by the children), that it lends itself to solution by one or more general problem solving strategies, that it often has multiple conditions and/or solutions and that the formal mathematics required is minimal. An example of this type of problem is presented below:

> Example B: I have $1.50 in quarters and nickels. They make 20 coins. How many quarters and nickels do I have?

Here there are six combinations of quarters and nickels which would total $1.60 and 19 combinations that would yield 20 coins. Of these 24 choices that satisfy either condition only one satisfies both conditions. The solution space for this problem is pictured on the next page.



Certain types of multiple condition problems, e.g., those with a low ratio of number of solutions satisfying all conditions to number of solutions satisfying some conditions, may be more readily solved than others. Yet, within some sort of reasonable limits, children observed during the 1974-75 year liked to at least try multiple condition problems.

Problem solving strategies. Another factor considered in the preliminary investigation of the central hypothesis (problem-solving performance can be improved by solving problems) was the set of strategies that might be focused upon. The decision concerning the strategies would affect the selection of problems. Two papers* commissioned by the project and reported in the 1975-76 proposal focused, among other things, on identifying and discussing strategies typically used in mathematical problem solving and/or strategies most appropriate for use by children in the intermediate grades. In addition, while search of the literature** indicates a diversity of problem-solving strategies, there are many common strategies. Which strategies ought to be included

* Dale Seymour, "Considerations for Mathematics Problem Solving Curriculum for the Intermediate Grades," Unpublished paper commissioned by the Mathematical Problem Solving Project, 1974.

Carole E. Greenes, "Identification of Problem-Solving Strategies," Unpublished paper commissioned by the Mathematical Problem Solving Project, 1974.

** Norman Webb, "A Review of the Literature Related to Problem-Solving Tasks and Problem-Solving Strategies Used by Students in Grades 4, 5, and 6," Unpublished paper commissioned by the Mathematical Problem Solving Project, September, 1974.

in the work of children in the intermediate grades remains an open question. Issues of desirable curriculum content as well as developmental ability are clearly involved, but the direction to follow is not easily seen. Even with the appropriate strategy types identified, the choice of specific problems remains open. This choice becomes especially critical if one goes back to the earlier statements regarding a problem and problem solution. If a problem statement is interpreted in various ways by various individuals and the respective interpretations are susceptible to varying solution modes, can a problem statement be written to elicit use of a particular strategy from children of this age? Is that even a desirable goal? Or, should a problem be such that it is potentially solvable by utilizing one of several strategies with less concern given to any particular one? In spite of the number of questions raised, it was judged that the process problems hold the greatest potential for eliciting problem solving in its broader sense and for eliciting strategies.

Classroom delivery format and other concerns. In addition to the problem types and strategies factors related to the hypothesis that children's problem-solving performance will be improved by solving problems, the practical issue of delivery into the classroom remains. Can one, as the hypothesis suggests, simply give children mathematics problems and ask the children to solve them? Given the definition of "problem", it would be prudent to inquire how an atypical (unknown) path to the solution is to be found without instruction? Would something like hints be of help? If so, would giving hints defeat the intent of the hypothesis? Another possible tack can be found in the learning by discovery literature. Can problems be structured so that students discover the path to solutions during the course of their

work? Such discoveries could come from hints, or, they could develop out of the problems themselves. A sequence, say, easy-to-hard, of problems might gradually develop an idea or, at least, gradually develop an attitude in students toward working on a problem they cannot solve immediately. Yet, how to sequence the problems is only one part of the classroom issue. How critical are the roles of the teacher and the students? If problems are simply given to the students, might they work together or would they work individually much like a test? With hints, one might ask if it would be the teacher who would hold them and on what basis they might be given out. In a learning-by-discovery system, students and teachers would necessarily be actively involved in a process of active learning and questioning.

Three factors considered in the preliminary investigation of the central hypothesis that problem-solving performance is improved by solving problems have been discussed. These factors are the problem type, problem solving strategies and the format for delivery. A further analysis of these factors is described in the following section.

B. *Observation of Children's Problem Solving*

At Indiana University three members of the Mathematical Problem Solving Project who had had considerable experience with children were assigned to develop and implement materials around the central hypothesis that children can become better problem solvers through solving problems. Prior to the actual development of these curricular materials, several questions were raised to which attention was given. These included:

1. Can a group of problems which elicit the desired strategies be identified?
2. Can the identified problems be placed in an easy-hard sequence?
3. Can good hints for the problems identified be found?
4. For the identified problems what materials seem appropriate?

5. How do children go about solving problems?
6. What is the role of the teacher in the problem solving session?

The best way to find the answers to these questions was to test various problems and procedures with children. The discussion which follows outlines some of the steps taken to answer the above questions. From the list of strategies that were suggested in the literature, the MPSP decided that the instructional materials would focus on the following strategies:

Making a list - organizing a list
Making a diagram (drawing a picture)
Looking for patterns
Estimate and check

With these strategies in mind, the team selected problems for the children to solve. The selection of problems was based on the following criteria:

1. The likelihood of eliciting one or more of the mentioned strategies.
2. The problem was placed in a context that a child would like and provided a task that a child would enjoy doing.
3. The likelihood that the child would experience to some extent, success.

An example of a problem that meets these criteria is:

> Jim works in an ice cream store. Ice cream cones cost 25¢. Show the ways that one could pay Jim for an ice cream cone.

This problem could be solved by making a list and possibly organizing it. The setting of an ice cream store interests children and they enjoy finding ways to make change. The problem has 13 possible solutions and most students should experience some degree of success.

Setting

With the cooperation of the local school personnel, a fifth-grade classroom reflecting a representative range of social and economic status was selected for the project. The teacher selected two groups of six

students each containing students of high, medium and low abilities. Members of the team worked with each group for 20-30 minute periods one or two times a week (11 times in 8 weeks) in an extra classroom.

Procedure

From among the available options it was decided to initiate the activity by having one group work in pairs or individually (their choice) while the other children worked together as a group. An unlimited supply of paper was made available and the children were asked to write down as much detail as possible showing how they had worked with the problem. The children's work was then collected and kept in individual folders. In addition, each session was audio-taped and a few sessions were videotaped.

To begin each session a team member (session leader) read the problem and answered questions from the children concerning the problem statement. One copy of the problem statement was available if any student wished to read it. While the students worked, the session leader was available for questions, experimented with giving hints and initiating discussions. The discussions were initiated by the session leader when the students reached a point when it was felt that the group should talk about the problem or when the problem was solved. During the discussions the students shared methods that they had used in attempting to find a solution. When the discussions had ended, the students' work was collected and another problem was presented or the group adjourned until the next meeting.

The other member(s) of the team served as observer(s). Particular attention was paid to how the students worked and to the role that the presenter played. Following each session with the children the team analyzed the data.

After 11 sessions with each group over an eight-week period, the decision to terminate the problem-solving sessions was made. Enough data for the pre-curriculum development had been collected. The results of the problem-solving sessions are summarized below.

1. Can a group of problems which elicit the desired strategies be identified?

Problems could be written which would elicit specific strategies. But the implementation of those strategies by the children was at varying levels of sophistication. For example, a child might make an estimate and check it. But the next estimate did not necessarily follow a discernable course of refinement. Or, a list might be made but no apparent pattern or organization was used in developing the elements of that list. However, regardless of the level of performance of a given child, the problems did elicit reasonable attempts at or approximations of the desired strategies.

The problems identified seem, in retrospect, to have some discernable characteristics. The problems were chosen because "they worked" and the characteristics were identified later. No checklist or algorithm for writing this type of problem is intended or felt to be possible at this time. The characteristics of these problems include:

(1) All the problems are placed in a context to which the children could and wanted to relate to. Some problems were based on a "real" situation; others were not so real but still of interest. The same math content in another context might well get little attention. For example, "How many ways can you make change for 25¢" generated less interest than the ice cream store problem mentioned earlier.

(2) None of the problems selected could be solved directly by an algorithm (at least not by one known to the child). This is in contrast

to the typical textbook problem which can usually be solved directly by application of an algorithm (or a combination of algorithms) such as the addition, subtraction, multiplication or division algorithm.

(3) All of the problems require minimal mathematical skills. At their simplest level, the problems could be solved by counting. No higher operations were needed and observations indicate that counting was used most frequently to solve the problems. For example, in the $1.60 problem the children could count by fives (often on their fingers) instead of multiplying or dividing by five.

(4) Most of the problems had multiple solutions or required finding several solutions to arrive at the final solution. For example, "Show all the ways John could get his allowance of $1.60 if he only got quarters and nickels" is a multiple solution problem. Asking, "How many ways could John get his allowance of $1.60 if he got quarters and nickels" is a single solution problem which, in effect, requires finding several solutions.

(5) Many of the problems included two or more conditions which had to be considered simultaneously. Some problems requiring the manipulation of two conditions were quite within many children's ability range. For example, in the $1.60 problem using quarters and nickels the children had little difficulty in organizing, to some extent, a list showing at least some of the solutions. However, those problems with three or more conditions were generally too difficult to elicit a particular strategy. The problem where John is paid his allowance of $1.60 in nickels, dimes and quarters with 19 coins is an example of a problem with (at least) three conditions. Here the solver must work with three instead of two different coins equaling $1.60 in addition to the third condition on using 19 coins. Some children did find solutions to this problem, but

did not solve it using a listing strategy. Other children would focus on one or two conditions while ignoring the other conditions.

(6) There was a conscious attempt to write the problems using language that would be correctly interpreted by children. While no formal criteria were developed for these characteristics, a strong "feel" for "good" problems grew and it became evident that problems which ought to elicit a particular strategy, from an adult's standpoint, would not necessarily elicit that strategy when the children solved the problem.

A note should be added here that these problems produced the desirable result that the children enjoyed working on the problems to the extent that they were anxious to get more and even asked to take work home. This highly motivated behavior is in contrast to the somewhat reticent outlook encountered at the beginning of these sessions. While it is clear that a part of this change must be attributed to the special nature of the situation and the relationship developed between the children and the MPSP personnel, it became equally apparent that a substantial part of the motivation was due to the interaction between the students and the problems. These children liked to solve these problems!

2. Can the identified problems be placed in an easy-to-hard sequence?

In order to develop an atmosphere conducive to open and imaginative attempts to solve problems within the groups, the initial sessions concentrated on interpersonal dynamics and the identification of "good" problems. Difficulty and sequencing were secondary issues. When, in about the sixth session, sequencing was addressed as an issue, it was discovered that the issue had changed.

An analysis of the first five sessions yielded the realization that problems which might be considered "difficult" were given to students in the early sessions. Also, difficult problems had, to some extent, been alternated with easier ones in the spirit of "torpedoing". Carrying this initial sequence into the remaining sessions, the early findings were validated. An easy-to-hard sequence was less successful in involving the children than simply providing problems that motivated them. There appeared to be a need for sequencing but the sequencing included many important factors such as motivation and variability as well as an easy-to-hard order.

3. Can good hints for the identified problems be developed?

Over time, attempts to provide hints fell into two categories: One was the typical type of hint that one finds in mathematics texts taking the form of an information provider or reminder and presented as a statement. When given this type of hint, students tried to fit the information into their solution mode whether it was actually appropriate or not. Frequently, a supposed hint became a new problem in that the student ended up trying to figure out how or why anyone would think the "hint" would actually help. The statement seemed to carry with it an imperative for use. The second category of attempts to help students' problem solving gain momentum took the form of questions. The questions themselves provided little direct aid or new information. What they really did was to get discussion going among the students. The questions were evaluated. If one seemed of no value, students rejected it. No imperative for use seemed to be attached to the questions. In this catalyst role, "hints" in the form of questions seemed quite good at getting students through an impasse.

4. How do children go about solving problems?

The structure initially chosen for the problem-solving sessions was presenting the problem, working on the problem and discussing the children's work. It was felt reasonable to begin in this fashion and then modify the structure as the children's needs seemed to dictate. The problem presentation period was intended to get the problem-solving statement to the children and allow them to clarify their understanding of it. During this time which lasted only a few minutes, the children initially asked questions aimed at both problem comprehension and at finding or gaining approval of a solution mode. The latter questioning dissipated quickly when the session leader refused to provide this direct type of assistance. To some extent, the comprehension questions decreased in number also. This, it is felt, reflected the children's increased skill at comprehending problem statements though this conclusion remains tentative. While variations of presenting the problem were tried (e.g., having a student read the problem statement, having each child read his/her own copy of the statement silently, having no presentation time as such) the existence of a "time" for presenting the problem and having comprehension questions asked seemed essential.

The period for working on the problem was, at the outset, the most open-ended of the three periods. More than with either of the other two periods, it was felt the children would have to provide their own path. Paper was given and grouping arrangements were made. From that point on, the children were in charge of their own efforts. As it turns out, no other formal structure was needed. While the team presenter remained in the general vicinity of the problem solvers, the children worked very much on their own. Most children varied the level and nature of their interaction with others. At times, some would work alone while remaining

(physically) within the group or the pair. Other times, a child might just look at another's paper. Looking at how other children solved a problem seemed to be a strong catalyst in helping a child get started or renew his/her efforts. Questions were posed to partners or to group members. Declarations were made and attacked. Arguments developed and were resolved. The flow of the interaction was rapid, varied and unpredictable. One child might sit quietly observing a discussion and suddenly cut through the difficulty with a resolution. Or through an insight, a debater might suddenly discover the "key" to the problem. The pace and nature of these problem-solving times varied. But, after several attempts at manipulating the structure of this period, it became clear that the children could and should be left to their own devices as long as behavior remained within reasonable limits. When the limits were crossed, a quick word from the session leader brought the children back to more acceptable behavior. Finally, it should be kept in mind that, on occasion, the children did ask the session leader questions. The availability of this outlet seemed indispensable and the manner in which it developed is discussed below.

The discussion session was intended to review and analyze the process of solving the problem and draw generalizations from the work done. Also, since no indication of rightness or wrongness of their solutions had been given to the children, it was felt that if such indication was indeed needed, this would be the place. As it developed, the discussion did all of these things to some extent. The strategies which the children employed were discussed. Generally, the children focused on the actions that led to their solutions. While many children were anxious to relate what they had done, others were not. And those who wanted to talk about their work were not necessarily the best problem

solvers and did not necessarily have the correct answer(s). Analyzing the strategies employed and generalizing took place only if initiated and directed by the session leader. While the students became better at analysis, e.g., looking for patterns, they did not initiate the activity. Generalizing to a formula remained a part of the discussion that met with little success or improvement. Although for some problems the children were not told the correct answer(s), they indicated a definite preference for being told the answer(s) after they had completed their work on a problem. No data other than student comments specifically support or refute the propriety of giving the answer(s), but since giving answers was most comfortable for the MPSP personnel and the children, this was done during the discussion.

5. What teacher actions are appropriate to complement (foster) the children's problem-solving behavior?

Since the children's activities in solving problems provided the lead for determining teacher action, these latter actions are covered in the three parts of the problem-solving session just outlined. The session leader was serving as a teacher in the sessions with children. An analysis of the session leader's actions was important so that later teachers might be advised.

The problem presentation period was simple and straightforward. The session leader read the problem and answered questions specifically relating to the meaning of the problem statement. The attempt was made in the reading of the problem to emphasize or otherwise signal important words, content, etc., and the problem was reread on request. Questions like, "Does that mean you're supposed to add?" or "What should I do?" were responded to with a statement to the effect of "I can't answer that". The children quickly accepted and adapted to this rule.

The role of the session leader during the working-on-the-problem phase, initially, was less clear. As mentioned, efforts were made to provide hints. Also, for a couple of sessions, children were allowed to ask any question they wanted except the "Is this the right answer?" variety. Questions like "What do I do now?" came up but nothing more substantial. Initially, "Is this right?" questions were asked too. Session leader moves of giving hints, answering any and all questions and not doing anything failed to foster children's progress in solving problems. Two session leader actions were gradually uncovered that did help. 1) Asking questions to foster comprehension or problem attack seem to move the children from dead center. For example, "What is the important information?", "What will the answer look like?", and a prod to searching out alternate solution modes, e.g., "Is there another way to try this problem?", often helped get a problem-solution process going again. 2) Forcing the question back into the group was another adult action that met with some success. Saying something like, "It might be a good idea to ask Frank", or "Have you checked with Karen on that?", tended to get the interaction going again. As discussed, this interaction then served to move the process forward.

Discussing the problem after the children had solved the problem was, like presenting the problem, more adult-centered and structured. The session leader organized and initiated the discussion and gave the answer(s) to the problem. As mentioned, children were eager to tell what they had done. As time went on, there were volunteers as soon as the discussion period began. Analysis and generalization required step-by-step questions from the session leader. Often the children did not seem to understand the line the session leader was following. In these cases, analyzing and generalizing was just dropped. As

mentioned earlier, the children wanted, in the end, to know the answer(s) to the problems. It was found that saying something like, "Here's how I'd solve this problem" and modeling a solution mode was most successful. At this point answer(s) was(were) given. This modeling was not a question-and-answer time. Nor was it a lesson or lecture. Rather the leader tried to discuss (show) how one might solve the problem. This modeling of a solution mode seemed to hold the children's interest best and it received the most favorable response.

Other Experiences

The team had additional experience with fourth and fifth graders by piloting evaluation instruments and other problem-solving materials that were being developed by the MPSP. The benefit of this work was two-fold. 1) The team gained experience from a classroom teacher's point of view by using various instructional modes with total classrooms. 2) As part of the evaluation instrument development of the MPSP, the team participated in piloting the instruments with various classrooms. One of the efforts involved giving a set of problems to an entire, intact classroom from which the children had been drawn for the special sessions. An informal attempt to sort the work of children who had been given special sessions from the other members of the class was made. It was possible to separate their work from their classmates based upon the ability to generate and sustain an attack on the problems. This informal sorting was not conclusive but it did provide an indication that solving problems might improve problem-solving ability.

C. *Preparation of Materials*

The goal of the problem-solving sessions and the other work with children had been to provide sufficient experience for the team so that problem-solving material based upon the central hypothesis could be

developed. The work with children had provided the critical elements for producing materials which would improve children's problem-solving abilities through solving problems. The elements include:

1. Creation of a classroom situation conducive to problem solving. To do this it was felt that the teacher had to facilitate students' efforts by:
   - legitimizing all students' attempts to solve problems,
   - aiding students only in understanding the problems,
   - allowing each student to follow his/her own sense of how to attack the problem,
   - not becoming involved in the problem-solving process per se and,
   - achieving closure on respective problems through a discussion including review, analysis, generalization and modeling.
2. In place of hints, questions which regenerated the group process were to be used.
3. The opportunity to view samples of others' work was thought to be useful.
4. Each student had to have control within wide deportment limits of his/her own problem-solving efforts.

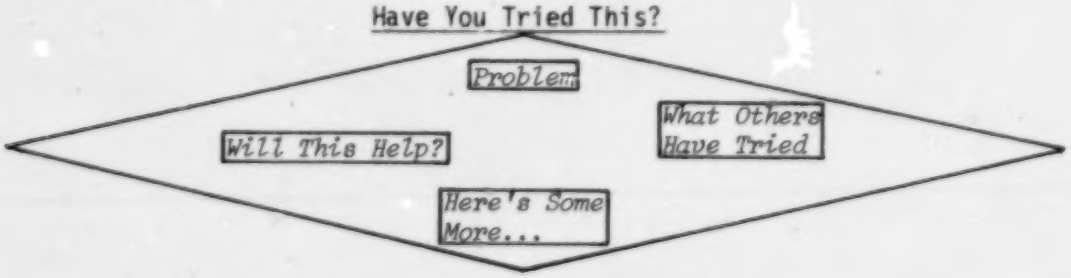
The focus of materials development had to be centered on interaction among the teacher, the students and the materials. The relatively novel yet necessary teacher behaviors were to be interfaced with the equally novel student behaviors. Yet there could be no massive in-service training of teachers. A way had to be found to present problems to children and to utilize appropriate teacher moves in the spirit of the conditions cited above. These factors resulted in the decision to develop a problem-solving bulletin board which, via a problem-a-day approach, seemed to have a good chance at being a possible vehicle to deliver problems to

the children without a great amount of work on the teacher's part.

The bulletin board contained four instructional parts. These parts of the board include:

1. "Problem"--located in the center at the top and is the statement of the problem.
2. "What Others Have Tried"--located below and to the right of the problem statement. It contains samples of other students' work on the problem.
3. "Will This Help?"--located below and to the left of the problem statement. It contains a group of questions and ideas that helped students in the problem-solving sessions progress with their work on the problem.
4. "Here's Some More...Interested?"--located in the center at the bottom contains additional problems--extensions of the problem statement for students to work on. This section was added to encourage student initiative as well as allow further study of problem-solving behavior.

Permanent holders were designed to hold the instructional parts for each problem. The permanent holders were placed on an orange diamond background so that it looked like the following:



The title page for each of the holders was designed to cover and hold the information for each part. The cover for the problem statement was made of clear acetate so that the problem of the day was always visible. The covers for the remaining parts were made of black acetate so that the

student had to lift it up to use the information. The title for each part was silk-screened on the acetate sheet in yellow. The use of orange, yellow, and black as well as the appearance of the acetate made the board a very attractive element in the classroom.

While the problem statement was always visible, the other parts of the board were designed so that the students could receive only one piece of information at a time. For example, the questions for "Will this help?" were typed on individual sheets of paper so that if the student wanted more information he needed only to lift the sheet of paper to find the next piece of information. The bottom of the pages were graduated so that it was easier to lift one sheet of paper each time.

The materials for each problem were stored together in a packet for the teacher. Each packet consisted of four pieces of materials which correspond to each of the sections of the board described above. To begin a problem, the teacher took the problem packet and placed each problem section in its place on the board.

Classroom Procedure

A classroom procedure for implementing the delivery system was developed. This procedure contains three steps: 1) Problem Introduction, 2) Solution Effort, and 3) Problem Discussion. Table 1 provides an overview of the steps and a listing of the possible teacher and student behaviors during each step. These listings are examples of behaviors to provide a feeling for each step. They are not intended to be complete or to represent the outer limits of behaviors for the respective steps.

Table 1

Bulletin Board Classroom Procedure

Step	Outcome	Possible Teacher Behaviors	Possible Student Behaviors
1. Problem Intro.	Students will understand problem statement	- put problem sections on boards - present problem to class - facilitate discussion/ question asking re: problem comprehension - organize problem-solving process - explains/reviews board sections	- read problem on board - listen to problem being read - write out problem - record key information contained in problem - ask questions to clarify problem
2. Solution Effort	Students will develop and try at least one strategy for solving problem	- devote class time to work on problem - facilitate student efforts - encourage students to help each other and/or share ideas	- works alone on problem - reads at least one section on board - works with at least one peer on problem/extensions - discusses problem with peers - works on and/or discusses problem/extensions with family members
3. Problem Discussion	Students will discuss their respective efforts and gain new insights into the problem	- devote 5 minutes to presentation of problem solution - allow students to demonstrate their (different) solutions/ processes - aid in bringing to light generalizations inherent in work on problem analysis - analysis	- offer to discuss own process/solution(s) - listen/discuss other's process/solution(s) - analysis - generalization

D. *The Pilot Study: Part I*

While the problem-solving board concept seemed intriguing, the lack of experience with this type of instructional material caused many questions to arise. It seemed prudent to pilot test, on a small basis, the instructional materials themselves before the materials were utilized to test the central hypothesis. A two-stage design was adopted to allow testing first of the bulletin board concept and second, assuming success of the pilot, of the central hypothesis (utilizing those materials).

The pilot study of the bulletin board materials focused on their viability by asking the following questions:

Does the problem-solving bulletin board:

1. generate the desired affective set?
2. stimulate the desired interaction among students?
3. adequately deliver the information to the students?
4. contain enough information to allow students to make reasonable attempts at solving the problems?
5. contain materials the students understand and utilize?
6. yield the desired teacher behaviors?
7. provide problem-solving practice to the extent that it improves problem-solving performance (in the spirit of the commonly-held conjecture)?

In addition to the specific issue of problem-solving board viability, questions of what accompanying materials teachers might need and what (classroom) teaching techniques worked well (and not so well) were included for examination.

Design

Initially, it was felt that observations of the problem-solving board in use in the classrooms and interviews with the participating teachers and some of their students would provide sufficient data to evaluate the viability of the problem-solving board. Later, after the

experience of piloting the evaluation material (see page 17) it was decided to add a comparison, on a post-test basis, of the children who had worked the ten problems on the problem-solving board with one class of fourth graders who had no contact with this pilot test effort. Four problems were selected representing a continuum from standard textbook problems to a process problem (as discussed earlier). While this comparison was recognized as a nonformal and fairly crude test, it was felt to be a fair preliminary look at the potential effect of the problem-solving board.

One third-grade, four fourth-grade and three fifth-grade teachers volunteered their classes for the pilot study. All but one of the fifth-grade classes were located in one school which had a population most representative of the total school district. The other fifth-grade class was selected on the basis of availability. These classes, as is typical in the school district, were self-contained except for reading, and consisted of 22-28 children each. The teachers represented a range of experience and styles.

Each teacher was given a complete problem-solving bulletin board and ten problem packets. Each packet containing a "Problem", "Will This Help", "What Others Have Tried", and "Here's Some More...Interested?", was held together by a paper clip, and the ten packets were contained in a large manila envelope. The packets were placed in each envelope in random order.

In-Service Procedure

A one-hour meeting was held for the teachers after the school day at the school building from which the majority of classes came. (The one fifth-grade teacher from another school was given materials and information on an individual basis.) Teachers had been visited prior

to the meeting and given an overview of the materials. Following the meeting, those in attendance were asked if they wanted to volunteer to try out the new materials. Seven teachers attended the meeting. F[illegible] volunteered and three others were recruited at a later date. An overview of the meeting is presented below:

I. Introduction: The goals of the meeting were outlined as being the presentation of the problem-solving board materials and the volunteering of participants for the pilot study. Background on the project as a whole and the first semester problem-solving sessions were given at this time.

II. Description of the problems and their use: The three-step process was described and some ideas regarding classroom use were discussed. Specific mention was made of the need for the teachers to adapt the board to their own classrooms so that a realistic view of its viability could be gained.

III. Solving a problem: One problem was solved with an MPSP person acting as teacher and the teachers in the role of students.

IV. A question and answer session

V. Request for volunteers

Study Procedure

Over a 21 school-day period, four fourth-grade, three fifth-grade and one third-grade class worked ten problems on the problem-solving bulletin board. Teachers conducted their classes as usual integrating the problem-solving materials in the manner they saw fit. This yielded a variety of teaching styles using the materials (open-traditional classrooms; teacher directed, non-teacher directed problem-solving sessions) and a variety of problem-solving environments (some students took problems home; some were to work on the problems during math time). The

three MPSP people visited each class as scheduling permitted, and on occasion, took charge of a session to experiment with teaching modes. Informal discussions were held with the teachers whenever possible and each was interviewed following completion of all problems. In addition, students from each classroom were interviewed. The students interviewed represented the range of ability and interest levels found in their rooms.

Results of Pilot Study I

1. DOES THE PROBLEM-SOLVING BULLETIN BOARD GENERATE THE DESIRED AFFECTIVE SET?

The bulletin board proved quite effective in generating the desired affective set. The observation data clearly shows that the students were involved, interested and enthused during the problem-solving sessions. The data also indicates that the teachers were quite pleased and excited by the students' interest and success with the board. In interviews, teachers related many instances of children checking the bulletin board for a new problem as soon as they entered the room in the morning and of children's disappointment if a problem-solving session had to be cancelled because of scheduling conflicts.

Several teachers were quite surprised to find this enthusiasm demonstrated by children who typically did little work (and poor quality work) in class. That the children were able to attack the problems independently and with such vigor also surprised and pleased most of the teachers. Student interviews supported the observation and teacher interview data. The children told of their own pleasure and excitement utilizing the problem-solving board and some expressed disappointment that the board was no longer being used. It seems quite evident that a very positive affective climate existed in all participating classes.

2. DOES THE PROBLEM-SOLVING BULLETIN BOARD STIMULATE THE DESIRED INTERACTION AMONG STUDENTS?

Observation data corroborated by teacher and student interviews, indicate that there was a substantial amount of interaction among students during the "Working on the Problem" stage. In a manner analgous to the earlier problem-solving sessions with small groups, the level and type of interaction varied among students and across problems. Yet, there was always the hum of discussion and it was unusual to observe a period during which a disagreement over a solution or solution mode did not take place. It is important to note that while there was much interaction among the students the children remained on task and there were no disciplinary problems to speak of.

3. DOES THE PROBLEM-SOLVING BULLETIN BOARD ADEQUATELY DELIVER THE INFORMATION TO THE STUDENTS?

All three data sources agree that the problem-solving board proved to be a very viable means of delivering information to the students. Its design and color scheme were attractive and the flaps covering some of the information served to raise the children's curiosity. While the manner in which the packets of information were held under the cover flaps was found to be flimsy for the type of use given the materials, the basic design was indeed adequate for the task.

4. DOES THE PROBLEM-SOLVING BULLETIN BOARD CONTAIN MATERIALS THE STUDENTS UNDERSTAND AND UTILIZE?

The problem statements in the "Problem" section of the board, by general agreement of observers, teachers and students were quite understandable and useable.

The questions in "Will This Help?" section also proved to be functional and understandable.

The samples of other children's work contained in the "What Others Have Tried" section received mixed comments. Observers indicate the section was useable and the samples seemed to be understood by the children. The teachers, however, generally felt the samples were hard to read because of poor reproduction and not easily interpreted. Children interviewed said they could read and understand the samples but that, generally, the section was of little help. Examinations of student papers and analysis of the interviews led to the conclusion that while teachers did not necessarily understand (or like) the samples of student work, the children did. Further, while the children could perceive no direct aid being provided, the samples did help provide the feeling of control over the problem. In the end then, the "What Others Have Tried" section seemed to be functional and understandable.

"Here's Some More...Interested?" problems received little attention while the board was in use. Teachers indicated they gave little attention to this section and did not really understand its function. Student interviews clearly showed they did not use the section and a main reason given was not knowing what it was about. For whatever the reason, the functioning of the "Here's Some More...Interested?" section was not clear to the participants and the section never got enough use to be evaluated.

5. DOES THE PROBLEM-SOLVING BULLETIN BOARD CONTAIN ENOUGH INFORMATION TO ALLOW STUDENTS TO MAKE REASONABLE ATTEMPTS AT SOLVING THE PROBLEMS?

The answer to this question requires some extrapolation of the observation and interview data. This is always a risky and potentially incomplete business. Suffice it then to say that no one was able to suggest additional information to include on the board which would further enhance its performance.

6. DOES THE PROBLEM-SOLVING BULLETIN BOARD YIELD THE DESIRED TEACHER BEHAVIOR?

Although the experience and styles of the participating teachers varied, a uniform pattern of techniques evolved for the problem-solving board use.

All teachers read the problem statements to the students and responded to comprehension questions. Queries directed at specific solution modes, e.g., "Does this mean add?" or at answers, e.g., "Is it 8?" created some discomfort. All teachers quickly (by the third or fourth problem) developed responses which did not answer these questions and did not create discomfort for either teacher or student. By the end of the pilot test, this stage was going smoothly and as expected.

The second stage of "Work on the Problem" also created some initial uneasiness in the teachers. The unfamiliar feel of the new materials coupled with a degree of skepticism regarding the children's chances of success contributed heavily to this feeling. Yet, throughout the pilot study, the teachers remained apart from the students' efforts and redirected questions to either the board itself or to other students. Phrases like "I can't tell you if its right but you might check with Esmerelda to see if she agrees with you," and "Would it help if you checked the board?" were developed by all teachers. Having this type of phrase available and seeing the children's enthusiastic efforts had the effect of making the teachers more comfortable with and staying apart from the children's efforts. At the end of the study, this stage was also going as desired.

The third (and final) stage, "Discussion" was, like the preceding two stages, an uncomfortable one for teachers initially. They were unsure how much input they were to provide and whether or not the "right answer(s)" were really to be given. Not knowing the right answers added to the

uncertainty. As the pilot study progressed, the teachers were able to feel at ease with the review of the children's problem solutions. One or two of the children were able to successfully model a solution mode. Generally, the review consisted of checking work that children put on the board. While success with analysis and generalization was expected to be minimal and the teachers were told this, they also agreed that attempting these activities was important. Yet, once the review of the children's work was complete, no one was able to make reasonable efforts at analyzing or generalizing. A question like, "Does anyone see anything else in this problem?" would receive no response and the matter was left. Further, the teachers were unable to achieve what the observers felt was satisfactory closure. Generally, the problem-solving session ended with the commencing of another lesson. A summary of a given problem was seldom done and when attempted, was done poorly. It must be kept in mind, however, that this limited realization of the Discussion stage did not affect the other two stages of the problem-solving process in a discernable way. While attention and effort must be given to improving the Discussion stage, caution must be applied so as not to dampen the affective set developed and lose the open approach the problem-solving board is able to generate.

The issue of teacher materials became the most frequently and vehemently addressed issue examined. From the in-service meeting through the end of the interviews teachers unanimously stated a desire for, at least, the answers to the problems. An interesting pattern related to this desire was described by these teachers. During the first few (2-4) problems, the teachers felt a great need for answers. Gradually, this need diminished and, while the request for answers remained, few specific reasons were given for the request and no other materials were requested.

7. DOES THE PROBLEM-SOLVING BULLETING BOARD PROVIDE PROBLEM-SOLVING PRACTICE TO THE EXTENT THAT IT IMPROVES PROBLEM-SOLVING PERFORMANCE?

The post-test comparison of work on problems done by participants in the pilot study with those who did not participate supported the earlier finding that solving problems in the manner described does improve problem-solving performance. By examining the post-tests to see whether the children's work (on paper) would demonstrate an ability to note pertinent problem characteristics and utilize at least one of the appropriate strategies, members of the Indiana staff not directly involved in the pilot study were able to identify 64% of the children who participated in the pilot. In contrast, only 27% of the non-participating children were identified as demonstrating these abilities. This identification was informal in the sense that no statistically reliable criteria exist. Yet, the consistent differentiation of the work on paper of the children who had been involved with project problems from those who had not continues to provide a strong suggestion that solving problems improves problem-solving ability.

ANALYSIS/DISCUSSION

The central goal of the pilot test was to test the viability of the problem-solving bulletin board. The goal was accomplished: The problem-solving bulletin board was shown to be a viable means of delivering problem-solving materials to the students in a manner that yielded open and imaginative efforts at solving problems. Sufficient information was presented in an appealing, understandable and useable fashion that students interacted with each other and their teachers in a desirable and productive manner. Data regarding teacher behavior was less abundant and, therefore, not as conclusive. Since the desired student behaviors were seen, one can to some extent assume that the desired teacher behaviors also occurred. Yet the data that does exist suggests that the Discussion stage of the

problem-solving process might not be as well developed by the teachers as it might have been. Clearly, a better picture of teacher behaviors is needed before any solidly based recommendations can be made.

It seems clear that teacher materials, in the form of answers, are needed. Although the need, on the part of the teachers, for materials decreased during the pilot study, the availability of answers seems to be a necessary (and reasonable) form of security for the teachers. It should be noted that with a minimal (1 hour) amount of in-service work the bulletin board and no teacher materials did yield, with the possible exception of the Discussion stage, the desired teacher behaviors.

The mechanics of the problem-solving bulletin board worked smoothly. The packets were easy to put up and take down. Storage of packets not in use was somewhat inconvenient and awkward. A change seems indicated. The flaps over the various sections of the board worked well though the strips of acetate holding the packets under the flaps do need to be secured more firmly. While it is clear the problem-solving board worked well, how it was utilized is not known. Questions like which sections were most utilized and whether certain problems generated more use remain to be answered. In a sense, the next level of questions about use of the problem-solving bulletin board were ready for examination.

Finally, and although the pilot was not primarily designed to look at student change, the differentiation on the basis of work on paper of the children who participated in the pilot study from those who did not provides an intimation that the type of open and imaginative effort at solving problems provided by the problem-solving bulletin board does, indeed, improve problem-solving ability. This preliminary finding suggests a rather powerful potential for this type of learning and certainly suggests the fruitfulness of pursuing the inquiry into both the problem-solving board and the hypothesis that children's problem-solving ability is improved by solving problems.

Comments

All efforts to this point had been directed toward a formal test of the hypothesis that solving problems will improve children's problem-solving performance. Work had progressed well and a suitable set of materials had been developed. While some questions about the problem-solving bulletin board did exist, they could be answered in the testing of the hypothesis. All systems were go except one. At this point notice was given that funding for MPSP would be terminated at the end of the academic year.

It was immediately clear that there was not sufficient time available for a thorough test of the hypothesis to occur. Something less comprehensive would have to occur in the remaining time. It was decided (after some deliberations) to attempt to gather the information about the problem-solving bulletin board to fill in the gaps from the pilot study. A more formal test of the central hypothesis would have to be delayed for another time and another project.

E. *The Pilot Study - Part II*

Part II of the pilot study was designed to examine in greater detail the workings of the problem-solving bulletin board related to the central hypothesis that problem-solving performance will be improved by solving problems. Specifically, the second study focused on the following questions:

1. Does the problem-solving bulletin board yield the desired teacher behaviors? Special attention to the Discussion stage of the classroom problem-solving procedure was essential to confirm or refute the rather sketchy data from the first study. Further, a clearer picture of the Discussion stage would permit a better study of the whole of teachers' behavior and the classroom problem-solving procedure.

2. Are answers sufficient as teacher materials? This issue follows directly from the first study and seems self-evident.
3. What use is given to the parts of the problem-solving bulletin board? While the first study demonstrated that the children did use the bulletin board, no data was kept as to the use which the respective parts received or to any patterns of use--of either the parts or the whole--that may have developed. Counts of the use of the respective parts of the board over the course of a study could provide valuable information on the effectiveness of the board and the growth of the children's skills.
4. Could insight be gained into the validity of the central hypothesis that problem-solving performance will be improved by solving the bulletin board problems? While it was fully recognized and accepted that no conclusive answer to the question of student growth was possible, an exploration of the issue was felt to be reasonable. Clues gained from this type of exploration might prove quite useful at a later date. Parenthetically, it might be added that attention to gaining some insight into the validity of the hypothesis was always present in the investigators' focus.

Materials

The bulletin board was kept intact with one change. The "Here's Some More...Interested?" section was eliminated and replaced with "What You Have Tried". This new section was a blank space outlined in yellow and was intended for student work. Those students who wanted to share their work with others had a place to put their work.

Teacher materials in the form of answers were developed. A single sheet for each problem was written. The sheet contained the answer(s) for a problem along with the following reminders:

1. Make sure everyone understands the problem.
2. Working on the problem is more important than getting a solution. Let the children "mess" with the problem.
3. Encourage children to talk with each other and work together if they want.
4. Allow and encourage as much use of the bulletin board as possible.

Also, all problem packets were placed in a sequence in a three-ring binder.

Finally, the number of problems was expanded to fifteen. Fourteen of the problems were chosen as seven pairs with each pair of problems being judged equivalent. Pairing the problems was done to facilitate an unobtrusive examination of student change.

Design

Essentially, the design that was utilized for the first study was utilized here. Observations were coupled with interviews of both the teachers and the students. In this study, teachers were scheduled at their convenience for use of the board and a more detailed observation protocol was developed.

Two additional items were added to this design: First, for each problem in the lower right-hand corner of each sheet of "Will This Help?" and "What Others Have Tried" a grid for students to check was placed. Students were to check each time they read a sheet. A tally of the checks would provide the needed data to analyze in detail use of the board.

Secondly, the pairing of the problems was intended to provide a basis for examining student change. Problems were paired so that numbers 2 and 15, 3 and 14, 4 and 13, etc., were equivalent. Work on the paired problems was to be examined for change and improvement. The first problem was intended solely as a warm up and was selected for that purpose.

Sample

Six classrooms from three schools were used in this study. Two fourth-grade rooms were in one school. One fifth grade was in another and two fifth- and one sixth-grade rooms were in the third. The fourth- and fifth-grade rooms functioned in a fashion similar to the classes in the first study and also contained 22-28 children. The sixth grade was a math class in a departmentalized middle school. It had 16 students. Taken as a group, the six classes represented the range of characteristics in the school district. The teachers were representative of the range of styles typically found.

Procedure

The distance between the school buildings necessitated holding three in-service meetings. Teachers had been recruited prior to the meetings so, unlike the first study, the sessions were directed entirely at the teachers gaining an understanding of and feel for the problem-solving bulletin board system. Each session covered the same topics and each remained on an informal basis. All meetings lasted 45-60 minutes. The issues covered during the respective sessions were:

I. Introduction: The concept and goals of the problem-solving bulletin board were described and background on the study was given.

II. Description of the board: A detailed description of the problem-solving bulletin board, its parts and the problem-solving classroom procedure was provided.

III. Solving a problem: The first problem in the classroom sequence was solved by the teachers who assumed the role of students. The Indiana personnel played teacher.

IV. Question and answer: The Indiana personnel discussed any questions the teachers had.

Over a twenty school-day period, the six classes worked on the fifteen problems. Four classes completed all problems; one class completed thirteen; one completed fourteen. The teachers adapted the materials as they saw fit and conducted the problem-solving sessions at a set time each day. This permitted more frequent and more thorough observations of the problem-solving bulletin board system in action than the previous pilot.

Results

1. DOES THE PROBLEM-SOLVING BULLETIN BOARD YIELD THE DESIRED TEACHER BEHAVIORS?

The primary focus here was to be the confirmation and detailing of the sketchy initial findings that stages one (Introduction of Problem) and two (Working on the Problem) went as expected. At stage three (Discussion) the observation reflected a variety of behavior ranging from little more than review of the children's work to extensive analysis. Essentially, what was observed added to and confirmed the information from the first study. The teachers experienced a general uneasiness during the first two or three problems. This gradually diminished. During the first stage, the teachers experienced some doubts regarding how much information they should give out. As more problems were worked, the boundary was established and no difficulties were encountered.

The second stage, Working on the Problem, went quite well. The teachers were able to remain outside of the children's problem-solving efforts. Initially, the teachers had some difficulty in not answering questions directed at solution accuracy or solution modes. But as the students' successes in working on the problems became more evident, and as the teachers became more familiar and facile with the needed techniques, this issue was resolved. Phrases like, "I can't tell you that but you

may want to check it with Hermine" and "Have you checked the board?" became part of the teachers' repertoire and were quite successful in getting the children back into the problem-solving process. The teachers generally reported a decrease in student questioning during this time.

During the third stage, the teachers generated a discussion of the work the students had done. Generally, this took the form of having a few (4-7) students put their work on the board and discussing it. This "discussing" ranged from a mere check for accuracy to a broad analysis of strategies used. One of the teachers consistently developed this stage by looking for patterns in the respective problems and relationships between problems. This was part of her style and her children seemed fairly able to understand her actions. In drawing out these patterns, she primarily developed ideas that the students had raised in response to her questioning. That they did respond at all is noteworthy. Although all teachers had to review the problem statement when student work was being examined, one teacher placed special emphasis on this point starting with the fifth problem. (Prior to this, the Discussion consisted of having problems put on the board and checked.) He constantly related work on the problem to the conditions of the problem. He also began modeling a solution mode in the manner suggested. During the modeling he also emphasized the necessary match between his actions and the problem constraints. These latter two teachers clearly made an attempt to utilize the Discussion section in a more meaningful way. One did so in the manner described during the inservice, the other in a manner more to his own style and unique to this point. Both methods seemed to involve a reasonable portion of their respective classes and, to this extent, were successful in enriching the discussion stage of the problem-solving process.

While there was a range of teacher behaviors in using the problem-solving board, it is instructive and of equal import to know that all teachers felt quite comfortable with the materials and all were able to adapt the problem-solving board to their own teaching style.

2. ARE ANSWERS SUFFICIENT AS TEACHER MATERIALS?

Providing answers to the teachers seemed to be enough teacher materials. The teachers stated that they referred to the answer sheets only during the Discussion stage, though a few did indicate they checked the number of solutions to multi-solution problems before the problem went up on the board. One teacher who began by reading the answer sheet before the problem went up stopped doing this because she was prone to give out all information she had (in her head). In effect, by not knowing the information, she was able to say "I don't know" and redirect the children to the bulletin board or their peers. Answers, then, provided the needed support and no teacher requested or thought of any other materials that would be helpful.

3. WHAT USE IS GIVEN TO THE PARTS OF THE BULLETIN BOARD?

The check-off system in the lower right-hand corner of the sheets on the problem-solving board failed in gathering data on use of the board. Teachers, generally, did not remind the students to use the grids. Where reminders were given, the instructions varied from reminder to reminder. Observations and interview data agree that use of the grids was inconsistent and erratic.

Observation reports show that with the first problems, the problem statement was probably the most used part of the board. Students read and reread the problem apparently to clarify their understanding even after the Introduction stage. Gradually, this concentration of energies seems to have dissipated to where the problem statement received less attention than the other parts of the board.

The overall use of the board diminished over time also. This was especially true when the second of the paired problems were done. The children commented as soon as the problem was presented that they had had one like that before. They were able to proceed with little attention to the board.

The function of the board seemed to vary from child to child. Some children invariably checked all parts of the board before working on the problem. Others checked it after their work was done. Some did not seem to use the board at all. Others used it only on some problems. (Whether those who did not use the board got the information by asking others is not known.) It seems that not only is a better record-keeping system needed but that some studies of individual students' use of the board would also be helpful in determining its actual function for the children.

4. COULD INSIGHT BE GAINED INTO STUDENT GROWTH AS A RESULT OF USING THE PROBLEM-SOLVING BULLETIN BOARD?

The design called for comparing students' written work on the paired problems. It was hoped that some preliminary ideas about changes in work as a result of working on the board could be gained. In fact, preliminary ideas were developed though not always in the direction anticipated. From the work on paper which was handed in, there was some discernable change from the first to the second problem of the paired problems. A small but consistent number of children showed some improvement in their work. This provides some small sense that growth indeed occurred.

A stronger feeling of student growth came from the observations and interviews with teachers and students. All of these sources agree that the children's behavior in each case on the second of the paired problems was grossly different than on the first. Immediately after hearing the

second problem, comments like "We've done this one before except the days are different" were made. Little attention was given to the board. Conversations with peers changed from abstract discussions of potential paths to matter-of-fact work with a known solution mode. In terms of the definition (see page 2), the second problem statement had ceased to be a problem. The path to the solution was known. That a problem type should become a painless exercise and not a problem after only one sample from the group is most impressive. Learning and retention at least for a 17-day period (this was the longest period between a pair of problems) most certainly happened.

For some children the first set of paired problems provided an adequate growth opportunity. No further growth was effected by the second set. For other children the second set of paired problems provided another growth experience. At the end of the problem sets it appeared evident that no additional growth would take place without some new influencing factor.

ANALYSIS/DISCUSSION

The second study succeeded in providing a more complete picture of teacher behavior in relation to the problem-solving bulletin board. This picture shows all teachers able to perform as desired for the Introduction and Work on Problem stages. The Discussion stage showed varying levels of development by the teachers. This is not a totally unexpected finding. Teachers are not typically required by most texts to lead the students toward analysis and generalization in the manner desired in the problem-solving bulletin board. Further, it is unreasonable to expect a one-hour in-service meeting to provide adequate training in such techniques. In spite of the variance in the Discussion stage, the desired student behaviors did occur and there is a strong indication

that growth also occurred. Any action to provide more substance to the Discussion stage should proceed with caution. Too much emphasis on it could dampen the overall effect of the problem-solving board. A balance must be achieved between theoretical thoroughness and empirical success. The key to this balance most likely lies in the students' excitement and enthusiasm in working with the problem-solving board.

Using only answers as teacher materials was successful. While it seems feasible that some additional materials might help ease the discomfort the teachers felt in the first few days, none of the teachers could identify any. They all agreed that the experience of those first problems was more instructive than any printed materials.

The failure of the check-off grids was a disappointment. Reliable hard data could have provided interesting and valuable insights into the use of the board and its component parts. In retrospect, the expectation that the children would conscientiously check the grid seems unrealistic. A more functional system needs to be devised.

Yet some valuable information, soft but valuable, was gained regarding use of the problem-solving board: The heavy rereading of the problem statement during the initial problems and the gradual tapering off of this activity is interesting. If one assumes the problems to be of roughly equal difficulty, it seems that the students became more able to pick up the information critical to the problem solution during the Introduction stage. Increased skill in understanding key issues in a problem statement is also suggested by the overall decrease in the use of the board. All other factors, e.g., interest level and success, being equal, the need for the assistance of the board seems to have diminished. That the children began looking at the board after they had worked the problem to their satisfaction suggests a continued need

for the board and a rather interesting set of learning styles.

It seems clear to the investigators that the central hypothesis that merely solving problems does improve problem-solving performance was substantiated at least as practiced in this study. Whether the three procedural stages (problem presentation, solution period, and discussion period) are essential elements, whether the particular set of problems or whether any of the other factors present in this study are essential remain open questions. A fuller investigation of the central hypothesis should be carried out. One may in turn ask if instruction can and/or should be employed to help the students move on to more sophisticated levels of thought and, if so, what instructional mode is best suited to do this.

F. *Summary Comments/Research Implications*

The effort described in this report represents a year-long attempt (1975-76) to gain insights into children's problem-solving efforts through an examination of the hypothesis that problem-solving performance is improved by solving problems. This section briefly summarizes the most important issues dealt with during this effort and provides some implications for future research suggested by each issue.

All the children involved in attempting to solve the various problems were able to employ, at some level of sophistication, the appropriate problem-solving strategies. This was done without formal instruction in the strategies. That children do employ the strategies opens the question of what strategies are employed most commonly by children and which are used most effectively.

The affect seen was generally and consistently high. The children enjoyed working on the problems and were motivated to make serious and sustained attempts to achieve solutions. The factors precipitating this response remain to be delineated and analyzed thoroughly.

The problem-solving bulletin board was successful. Students were able to employ the desired strategies and they were motivated to work on the problems. Affect was good. Further, the board was adaptable to a variety of classrooms and teaching styles. This initial success with one type of curricular materials strongly suggests the value of a broad scale development and study program on an ongoing basis.

Throughout the year, the different groups of students who participated in the efforts described herein demonstrated a leveling phenomenon that was roughly similar for respective grade levels. For example, generally speaking, the fifth-grade students who participated in the initial small group work, those involved in the first pilot study, and those involved in the second pilot study all reached the same level of sophistication in problem solving. The use of formal instrumentation to validate this finding is a needed step. Further, if the findings are validated as anticipated, the reason for this leveling phenomenon would remain to be uncovered. Following in this line are issues of what new materials and/or strategies could be employed to move the students beyond the leveled degree of problem-solving sophistication.

Finally, the central hypothesis that problem-solving performance can be improved by solving problems received strong, consistent, and continuous support throughout the year-long effort. Over two hundred students from three grade levels solved a variety of problems that employed a variety of problem-solving strategies. Formal instruction was never deliberately employed and yet all students, virtually without exception, improved to some extent in either or both the cognitive and affective domains. The fruitfulness of studying problem solving via exploration of this hypothesis is forcefully demonstrated by findings presented throughout this report and by the wealth of potential topics

generated by the exploration and sketched above. Although much work remains, a small but valuable step toward understanding and improving children's problem-solving performance was made through focusing on this hypothesis.

G. *Some Conjectures Related to Children's Problem Solving*

The report of the investigation on learning to solve problems by solving problems contained in the preceding sections was based in part on observations and conjectures made during the 1974-75 year of MPSP. The observations collected and conjectures made during that year formed the basis for many of the activities of the year 1975-76, during which this investigation took place.

In 1974-75 the observations of children's problem-solving efforts were made in three ways:

1. by watching individual children and groups of children as they solved problems;
2. through analysis of the children's written work;
3. through interviews with children regarding their work in problem solving.

The observation of the children was limited to fifth-graders so that developmental factors would play the least possible role.

Problem-Solving Sessions

Problem-solving sessions with the children generally were of two types: (1) observation of children working alone, and (2) observation of children working in groups of three or four. In a few instances, slightly larger groups were observed. It was anticipated that children might perform differently when they worked in groups than when they worked alone. The observers' role during these sessions was to present the problems to the children, to answer questions about the meaning of words or terms, to clear up misconceptions about the problems if they

arose, and to encourage the children to write down or talk about what they were doing or thinking. In most cases, the sessions were audio-tape recorded for later analysis. In addition the observer took notes to supplement the tapes. The children were provided with paper and pencils if necessary and any materials relevant to the problem. For certain problems concrete aids (chips, blocks, geometric shapes, etc.) were also provided. The observer showed the students what materials were available to them and urged them to use whatever materials they wished.

Among the observations initially made are the following:

- Children usually wanted to "compute" the answer. They seemed to be conditioned to add, subtract, multiply or divide.
- They were "answer"-oriented.
- They did not want to "mess around" with paper and pencil.
- They were not used to working in small-group settings cooperatively.
- They had not had experience with the process-type problem.
- They relied heavily on adult (teacher) guidance and direction.

On the other hand, with some time and encouragement, the children did begin to solve some of the process problems. From the above observations and subsequent actions of the children the following conjectures were formed.

Conjecture 1. Children (grade 5) can solve some process problems.

The observers analyzed the work of children in their problem-solving efforts. The children initially did not have many strategies to call upon. Most of the children made a "stab" at a possible answer and asked the adult if the answer was correct. When they received no response other than a smile and "What do you think?" the children turned

to check their own work or asked another student. Essentially no other strategy was observed.

Conjecture 2. Children (grade 5) without training use only a trial-and-error strategy in solving process problems.

An attempt to examine what made a problem difficult was pursued using different types of problems. Issues related to reading and word, phrase, and sentence complexity were examined by presenting the same problem using different words and sentence structure. Problem difficulty was also examined by varying the number of conditions. While the issues of reading and number of conditions seem to have a direct bearing on problem difficulty, the issue of problem interest or motivation seemed to have the greatest weight. In other words, if the children found the problem interesting, they generally could handle difficulties related to reading and number of conditions. The number of conditions was a factor in difficulty, but just where the difficulty exists is not clear. Sometimes children could solve a three-condition problem, but not a two-condition problem. There seemed to be an interaction among difficulty, number of conditions and the size of the solution spaces for each condition (i.e., number of possible solutions for each condition).

Conjecture 3. Children (grade 5) can solve some multiple-condition problems; but the difficulty of these problems seems to be related to the number of conditions, size of the solution space and the motivation inherent in the problem.

Toward the end of the first year, an attempt was made to teach some strategies to children to see if they could use such strategies. This teaching was done by asking the children to solve a problem and then discussing and analyzing it. In this analysis, different ways of solving the problem were presented using various strategies. The children were

encouraged to use some of these new strategies in solving these new problems.

Conjecture 4. Children (grade 5) can be taught some problem-solving strategies and they can use them in solving new problems.

The fact that children could use some strategies in solving problems encouraged the MPSP staff to seek a way to teach some strategies in a more realistic setting than using one adult with three or four children. Further, there was some question as to which strategies were more appropriate to teach the children. During the second year, efforts were continued to work with small group-teaching and observation (as described in previous sections of this report). The activities in the early part of the year indicated that children grew in problem-solving skill simply by solving problems.

Conjecture 5. An effective way for children (grade 5) to learn to solve problems is by solving problems.

The studies related to the problems presented on the bulletin board developed from conjecture 5. As the work with the bulletin board continued, the observations shifted from observing only children to observing both teachers and children. There were three phases in the bulletin board problem-solving study: the problem presentation stage, the work-on-the-problem stage, and the debriefing or discussing-the-problem stage. Students whose teachers spent time analyzing and generalizing problem solutions in the discussing-the-problem stage seemed to exhibit greater growth in problem-solving strategy acquisition.

Conjecture 6. The acquisition of problem-solving strategies by children (grade 6) seems to be related to the teacher-directed analysis and generalization of problem solutions.

The six conjectures listed above were gathered through observations over a period of two years. While the MPSP project staff hoped to gather more evidence to support (or reject) these conjectures, this hope was not realized under this grant. On the other hand, these conjectures can provide individual staff members (or other mathematics educators) a point of departure in their individual study of children's problem solving.

APPENDICES TO

TECHNICAL REPORT II: INSTRUCTIONAL MATERIALS

PART D: LEARNING TO SOLVE PROBLEMS
BY SOLVING PROBLEMS

APPENDIX A: Components of the Problem Solving Bulletin Board

APPENDIX B: Observation and Interview Forms

(Appendices Under Separate Cover)